LET TIEBOUT PICK UP THE TAB: PRICING OUT EXTERNALITIES WITH FREE MOBILITY

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Abstract

Free mobility has not been thought of as an effective tool to correct over- or underproduction of externalities. In this paper, we establish that foot voting can internalize the cost of negative externalities. Workers have to accept the wage and rent, however high or low these values are in equilibrium, *if* they cannot relocate. In reality, workers are mobile and they can effectively influence the equilibrium wage and rent to reflect the externalities by threatening to walk away if the current externalities are at an intolerable level. In an economy with free mobility, firms indirectly pay for the damage in the form of an increased labor or land cost and thus the externalities are partially internalized. We specify the condition under which a mobile economy is efficient in the presence of externalities, and discuss potential policy implications of our findings.

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JEL classification: D62, R23, R13, Q5

1 INTRODUCTION

Externalities are stressful to deal with. Decentralized decision making does not get us to an efficient allocation. Externalities, by definition, are not priced to reflect beneficial or adverse effects imposed on a third party. Competitive equilibrium assigns zero to the shadow price of the third party's objective function and/or constraint when these should not be slack. There have been many attempts to assign the right multiplier to the third party. Duranton and Puga [DP04] list four equilibrium concepts used in an urban economic context: competitive, free mobility, Nash and core. Just because the competitive equilibrium fails to deliver efficiency does not mean that the other three fail as well. We will show that under some conditions, the introduction of free mobility can Pareto improve upon the competitive equilibrium when externalities are present.

In Roback [Rob82], the firms take the amenity level as given. Here, we will let the firms pick their industrial emission levels (which correspond to the amenity

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level in [Rob82]) on their own. It is at their discretion to adopt pollution abatement measures and decide how much reduction in pollution is optimal. The tradeoff that the firms face is as follows: While abatement technology is costly to implement, the improved living environment realized through reduced industrial emissions will lower the wage and thus the firms can recoup the emission-related cost increase from reduced labor cost. The firms decide how far they can go with their emissions before the air pollution starts to eat into their profit through increased labor cost.

Various alternatives or fixes to competitive equilibrium have been suggested, including Pigouvian tax, tradable permits, bargaining ([Coa60]), or command and control, among others. We propose another alternative that we have already set in place without realizing it. The fixes listed above do not have a geographical dimension. For example, Baumol [Bau72] mentions the relocation of the laundry industry to get away from smoke, and Pigouvian tax is proposed to control the migration dynamics of the industry. The place that the laundry industry relocates to is presumably still within the same city as there is only one location in the model. The effects of smoky air on the quality of life, and consequently on the labor cost, are not considered. Residents cannot move out of their current location regardless of the level of externalities because there are no other locations available.

The international trade literature does treat pollution in a geographical setting. Merrifield [Mer88] discusses pollution abatement technologies in the international context. While pollutants and goods are internationally mobile, workers are not. The equilibrium utility level will not equalize across the border, and thus wages or rents do not work as a compensatory mechanism for differing living environments by location as established by Roback [Rob82]. Pethig [Pet76] discusses the location of production as a result of welfare differentials but not the location of consumers. In fact, utility maximization is absent in [Mer88], as transborder pollution is received by the producers in the form of reduced productivity. Forster [For81] talks about labor mobility but it pertains to industries, not locations. By and large, workers are not mobile in the international trade literature.

In reality, we usually have competitive outside options as individuals. If externalities are over- or underproduced to our liking, we can always pack up and leave to find another city that offers a more desirable allocation. While some externalities spread uniformly across the country, most of them are either fully contained within a limited area or decay rapidly with distance.[1] Unlike these location-bound externalities, consumers are not inexorably tied to a particular city. The footloose nature of consumers has been overlooked but deserves some attention because it does exist in reality.

Tiebout [Tie56] was the first to recognize that unconstrained residential choice of jurisdictions emulates the competitive equilibrium for local public goods under

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[1] This is the premise of aforementioned [Bau72], where the laundry industry can avoid the effects of the externalities by relocation.

certain tax schemes. We will expand on this idea to see if and when foot voting works to manage externalities, which are a generalization of local public goods. Our departure from Tiebout's work is that there is no public sector and externalities are created as a result of firms' profit maximization. We do not assign a nonzero shadow price on the injured party (in our case, consumers) per se, but rather, we will have firms realize it through the market equilibrium with free mobility. They are still welcome to ignore the damage they inflict upon the consumers but doing so will be costly because that can come with a higher equilibrium wage and/or rent and their profitability may suffer. Thus, we can use free mobility to price out at least some externalities, or, to put it in another way, let Tiebout take care of the bill for the shadow price.

In fact, compensation through the equilibrium rent or wage is not a brand-new idea. In the quality of life literature, it is known that residents in a subpar city in terms of living environment are compensated for the quality of life with a lower equilibrium rent and/or a higher equilibrium wage. In this light, we will base our model on Roback [Rob82] and integrate Tiebout's idea into it.

As such, there are two lines of research related to our work. One is on the quality of life and the other is public finance. The literature on the quality of life assumes that the amenity level is predetermined (Rosen [Ros79] without land as a factor of production and Roback [Rob82] with land as a factor). On the other hand, in public finance, the choice of public goods is usually endogenous but the scope of goods is limited to public goods.[2] Public goods are an intended consequence of production, whereas externalities such as pollution do not have to be intentional. Public goods can be thought of as a special case of our model, where a firm (or, more likely, the government) produces externalities on purpose.[3]

The equilibrium we present here may or may not be socially optimal depending on the magnitude of multiple factors involved. The case in point, however, is that intercity migration will alleviate the market failure and that we do not have to be overly pessimistic about the equilibrium allocation in the presence of externalities: It could have been much worse if workers were tied to where they were born.

In section 2 we introduce a production economy model to show that under certain conditions, firms in an economy with free mobility voluntarily produce less negative externalities than firms in an economy with no mobility. Section 3 discusses potential policy implications of our model, before section 4 summarizes our findings.

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[2] Scotchmer [Sco94] interprets public goods in a broader sense.

[3] In the case of the government producing externalities, their objective is not a profit (i.e., government surplus) maximization.

2 THE MODEL

To identify the role that free mobility plays in alleviating negative externalities, we construct a production economy with no labor mobility (immobile economy) and then compare it to a production economy with free mobility (mobile economy).

Consider an economy in some city, say Knoxville, populated by $J_k \in \mathbb{R}_+$ identical consumers. Josh is a representative resident of Knoxville. He is endowed with one unit of time, and consumes composite goods c_k and housing $\tilde{h}_k$. Composite goods are a numéraire and the rent is r_k dollars per unit of land. Labor supply is perfectly inelastic and Josh supplies one unit of labor for wage w_k. We assume that the allocation depends only on location so that the consumption bundle will be the same for any resident of Knoxville.

All the land is owned by an absentee landlord, whose function is limited to the provision of land for residents and firms. Land is interchangeable between residential and industrial purposes. A residential lot $\tilde{h}_k$ can be converted into a production site h_k for firms and vice versa at no cost. Land supply H is perfectly inelastic and thus the landlord's action has no impact on resultant allocations. We do not let Josh own the land so that the rent he pays will not correlate with rental income, which would otherwise blur the role the rent plays in coordinating the quality of life across cities as we will see later in section 2.4.

There are $I \in \mathbb{R}_+$ identical firms in Knoxville, operating in a perfectly competitive fashion. As with consumers, firms' optimization is assumed to be identical within each city. Pick a representative firm, say Ironworks Inc. Ironworks employs l_k hours of labor and lease their production site h_k for r_k dollars per unit. In addition, they are free to produce negative externalities $e_k \in \mathbb{R}_+$, which boost their productivity.[4] As customary in environmental economics (cf. Cropper [CO92]), we will take e_k as an input rather than an output. The production function is given by a $\mathcal{C}^1$ function $f(l_k, h_k, e_k)$. We assume that $f(\cdot)$ is strictly concave and exhibits constant returns to scale in (l_k, h_k) for each e_k they choose. In addition, we introduce the following assumptions:

ASSUMPTION 2.1: NEGATIVE EXTERNALITIES

Negative externalities e_k satisfy the following:

Concave and Single-Peaked. *For any $(l_k, h_k) \in \mathbb{R}^2_+$, $f(\cdot)$ is concave in e_k and single-peaked at $\bar{e}_k (> 0)$.*

Linear and Non-Rivalrous Emissions. *Each consumer registers aggregate emission level $E_k := I e_k$ as the negative externalities relevant to his welfare.*

No Inter-Firm Effects. *A firm's choice of e_k has no effect on other firms' productivity.*

No Cross-Border Effects. *E_k is determined solely by the firms operating in the same city.*

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[4] We take e_k as a scalar. It can be a vector of various negative externalities but the following arguments remain essentially the same. Alternatively, e_k can be thought of as an index of all sorts of negative externalities produced.

The first assumption is made with the understanding that the emission of pollutants improves productivity as it frees up the resources that would otherwise be earmarked for abatement; however, the air will be saturated with pollutants if they go above some level $\bar{e}_k$. Productivity starts to show signs of exhaustion past $\bar{e}_k$. Since all the firms employ the same production technology as Ironworks, $\bar{e}_k$ will take the same value regardless of the operator.

Second, the level E_k of negative externalities that Josh experiences is the sum of individual (and anonymous) emissions e_k. Industrial emissions are not rivalrous. Any resident of Knoxville faces as many emissions as Josh does.[5]

Third, Ironworks' emissions only compromise consumer welfare and do not inflict any harm on other firms in operation. The production function depends on e_k rather than E_k. Enhanced productivity achieved through *own* emissions e_k (or productivity loss if Ironworks goes past $\bar{e}_k$) is considered to be much greater than the change in their productivity caused by the *overall* emission level E_k, the ramifications of which are negligible to Ironworks in comparison. This assumption is included to isolate the effect of free mobility from a free-rider problem (cf. appendix A.1).

The last assumption implies that negative externalities stay within the city border. Knoxvillians will not suffer from emissions that originate from other cities. We will comment on long-range externalities in section 4.

Josh will take E_k as given. His preferences are represented by a $\mathcal{C}^1$ utility function $u\left(c_k,\ \tilde{h}_k;\ E_k\right)$ that is strictly convex in $\left(c_k,\ \tilde{h}_k\right)$. Assume $\partial u(\cdot)/\partial E_k < 0$ for any $\left(c_k,\ \tilde{h}_k\right)$ and E_k. Denote Josh's indirect utility function by $v(w_k,\ r_k;\ E_k)$.

As for the landlord, we assume that he will only enjoy composite goods and that emissions will not affect his welfare as he is an *absentee* landlord. He will simply rent out all the land H in Knoxville for r_k per unit and tap out his rental income thus earned to buy as many composite goods as possible in equilibrium.

2.1 MOBILE AND IMMOBILE ECONOMIES

We will see how restricted mobility will interfere with equilibrium allocations. Let $\mathcal{P}^{free}$ denote a mobile economy and $\mathcal{P}^{imb}$ an immobile economy. In $\mathcal{P}^{free}$ the utility level is so determined that any non-vacant city will achieve the same utility level in equilibrium. If there is any differential, then workers will relocate to seize an opportunity to improve their welfare elsewhere. On the other hand, in $\mathcal{P}^{imb}$ Josh is tied to Knoxville. Consequently, while Ironworks' decision alters the equilibrium labor supply in $\mathcal{P}^{free}$, it has no bearing on the labor supply in $\mathcal{P}^{imb}$.

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[5] The number of firms I is arbitrary. We assume constant returns to scale in $(l_k,\ h_k)$, and thus an industry can be divided into any number of firms. The firm size matters only when we aggregate individual emissions e_k into E_k. Due to concavity in assumption 2.1, e_k grows with I. We would like to keep externalities as our focus rather than organizational matters in the industry. We simply assume that I is the same across the country so that an (arbitrary) number of firms will not interfere with the allocation.

The set $\mathcal{E}^{free}$ of equilibria in $\mathcal{P}^{free}$ is a subset of its counterpart $\mathcal{E}^{imb}$ in $\mathcal{P}^{imb}$. In $\mathcal{P}^{imb}$, J_k is predetermined and Ironworks' action has no consequences for labor supply. An immobile equilibrium can be found for any J_k (as long as it exists). In $\mathcal{P}^{free}$, J_k cannot be a random constant. Some population distribution $J \in \mathcal{J} := \left\{(J_k)_{k=1}^{K} \in \mathbb{R}_+^K : \sum_k J_k = \bar{J}\right\}$ will fail to equalize the utility levels across the cities ($\bar{J}$ is the total population of the country).[6] $\mathcal{E}^{free}$ is more restrictive than $\mathcal{E}^{imb}$ because it has to constitute an equilibrium in $\mathcal{P}^{imb}$ and *in addition*, it needs to meet the extra constraint, namely, utility equalization across the cities (6) to be specified later in section 2.3.

We will compare an equilibrium that belongs to $\mathcal{E}^{free} \left(\subseteq \mathcal{E}^{imb}\right)$ to another equilibrium in $\mathcal{E}^{imb} \backslash \mathcal{E}^{free}$ to examine the role that free mobility plays in the presence of externalities.

For consumers and landlords, optimization is the same whether workers are restricted to Knoxville or not.[7] Mobility kicks in when we consider Ironworks' behavior. Let us consider an immobile equilibrium first.

2.2 Immobile Equilibrium

Ironworks maximize their profit

$$\max_{l_k,\ h_k,\ e_k} f\left(l_k,\ h_k,\ e_k\right) - w_k l_k - r_k h_k.$$

The first order conditions are:

$$\frac{\partial f(\cdot)}{\partial l_k} = w_k, \quad \frac{\partial f(\cdot)}{\partial h_k} = r_k, \quad \text{and} \tag{1}$$

$$\frac{\partial f(\cdot)}{\partial e_k} = \frac{dw_k}{de_k} l_k + \frac{dr}{de_k} h_k. \tag{2}$$

Ironworks will not factor in the social cost of emissions, in particular $\partial u(\cdot)/\partial e_k (< 0)$, in comparison to (16) in section 2.6, where we will compare the equilibrium with the efficient allocation.

Note that unlimited emission is possible but (implicitly) costly to make, and Ironworks does not necessarily go overboard on emissions even in $\mathcal{P}^{imb}$. Ironworks has to gauge the indirect consequences of emissions that reduce their profit in general equilibrium even when the citywide supply of labor J_k is fixed. Josh's marginal rate of substitution is contingent on the ongoing emission level and, as a result, the equilibrium prices are not independent of the selected emission level. In particular, while the length of leisure is fixed, housing and composite good consumption will be realigned against any change in the emission level. In turn, Ironworks will rearrange their production plan in response to changing factor prices. For instance, if

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[6] We will assume that $\bar{J}$ is exogenous so that any difference between the mobile and immobile equilibrium is fully ascribed to free mobility rather than population growth or immigration from overseas.

[7] Except that in $\mathcal{P}^{free}$ Josh needs to choose his location in addition to his consumption bundle.

an increase in emission levels leads to a reduced consumption of composite goods and increased housing consumption, the rent will go up. Ironworks will downsize their production site and reconsider their employment and emission levels in response. Furthermore, a change in the emission level alters the marginal product of labor. In the end, emissions do not necessarily come for free and blindly setting e_k at $\bar{e}_k$ is not a solution even when labor mobility is completely restricted. The condition (2) captures this process: The marginal increase in revenue from emissions should be offset by the marginal increase or decrease in rent and labor cost from emissions, the sum of which may or may not be zero.

The land and labor market clearing conditions are

$$I l_k(w_k,\ r_k,\ e_k) = J_k \tag{3}$$

$$J_k \tilde{h}_k(w_k,\ r_k,\ E_k) + I h_k(w_k,\ r_k,\ e_k) = H, \tag{4}$$

where $l_k(\cdot)$ is labor demand, and $\tilde{h}_k(\cdot)$ and $h_k(\cdot)$ are residential and industrial demand for land. For any given J_k, solve (3) and (4) to obtain $w_k = w_k(E_k;\ J_k)$ and $r_k = r_k(E_k;\ J_k)$. Furthermore, apply the implicit function theorem to (3) and (4) to find $dw_k/de_k = dw_k/dE_k = w_k'(E_k;\ J_k)$ and $dr_k/de_k = dr_k/dE_k = r_k'(E_k;\ J_k)$.[8] Then e_k solves (2) now revised to

$$\begin{aligned} &\frac{\partial f\ \{J_k/I,\ h_k(w_k[E_k;\ J_k],\ r_k[E_k;\ J_k],\ e_k)\}}{\partial e_k} \\ &= w_k'(E_k;\ J_k)\tfrac{J_k}{I} + r_k'(E_k;\ J_k) h_k(w_k[E_k;\ J_k],\ r_k[E_k;\ J_k],\ e_k). \end{aligned} \tag{5}$$

Given $J \in \mathcal{J}$, let $e^{imb}(J) := \left\{e_k^{imb}(J_k)\right\}_{k=1}^{K}$, where $e_k^{imb}(J_k)$ is the solution to (5) for each k.

2.3 MOBILE EQUILIBRIUM

Next, consider a mobile economy, where J_k is no longer exogenous. For simplicity, we assume that no city is vacant ($J_k > 0$ for all k). In a mobile economy, the equilibrium has to meet (3) and (4) so that it is at least a member of $\mathcal{E}^{imb}$. In addition, $\mathcal{E}^{free}$ requires

$$v_k(w_k,\ r_k,\ E_k) = v_1(w_1,\ r_1,\ E_1), \quad \forall k \in \{2,\ \cdots,\ K\} \tag{6}$$

$$\sum_k J_k = \bar{J}. \tag{7}$$

[9] Given $e := (e_k)_{k=1}^K$ with the corresponding aggregate emission level $E := (E_k)_{k=1}^K$, solve (3), (4), (6) and (7) for $(w_k,\ r_k,\ J_k)_{k=1}^K$ to obtain $(w_k(E),\ r_k(E),\ J_k(E))_{k=1}^K$. Apply the implicit function theorem to (3), (4), (6) and (7) to obtain $\partial w_k(E)/\partial e_k$ and

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[8] From assumption 2.1, $\partial E_k/\partial e_k = 1$ when one firm increases the emission level while the remaining firms do not.

[9] Utility functions do not have to be location dependent but we will add subscript k to $v_k(\cdot)$ as deemed necessary for tracking purposes.

$\partial r_k(E)/\partial e_k$. In $\mathcal{E}^{free}$, e solves

$$\frac{\partial f\left\{J_k(E)/I,\ h_k\left[w_k(E),\ r_k(E),\ e_k\right]\right\}}{\partial e_k} = w_k'(E)\frac{J_k(E)}{I} + r_k'(E)h_k\left(w_k(E),\ r_k(E),\ e_k\right). \tag{8}$$

for all $k \in \{1,\ \cdots,\ K\}$. Denote the solution by e^{free}.

2.4 Comparison between Mobile and Immobile Equilibrium

With the solution $e^{imb}(J)$ and e^{free} at hand, we can now compare $\mathcal{E}^{free}$ to $\mathcal{E}^{imb}$. Pick some equilibrium emission level e^{free} from $\mathcal{E}^{free}$ with the associated equilibrium population distribution J^{free}. Note $e^{imb}\left(J^{free}\right) = e^{free}$: If J^{free} already constitutes an equilibrium in $\mathcal{P}^{free}$, then the closure of city borders has no impact on the allocation because nobody wants to move either way. That is, when intercity flow of labor is obstructed, (3) and (4) are still satisfied, (6) and (7) are also satisfied but not necessary in $\mathcal{E}^{imb}$ anyway, and (5) doubles as (8). In this case, mobility makes no difference to emissions and by extension, utility levels. The end result will be the same whether Josh is location-restricted or not. We need to wiggle the population distribution to tease out the role free mobility serves: Consider a perturbation $\tilde{J}^{free} := \left(J_1^{free},\ \cdots,\ J_k^{free}+\varepsilon,\ J_{k+1}^{free}-\varepsilon,\ \cdots,\ J_K^{free}\right)$ for some small $\varepsilon(>0)$ to swing the resulting equilibrium allocation out of $\mathcal{E}^{free}$. Denote the new equilibrium emission level $e^{imb}\left(\tilde{J}^{free}\right)$ by $\tilde{e}^{imb}$, which is a member of $\mathcal{E}^{imb}$ but no longer of $\mathcal{E}^{free}$. Since $\tilde{e}^{imb}$ thus derived only violates (6), any difference between $e^{free}\left(= e^{imb}\left(J^{free}\right)\right)$ and $\tilde{e}^{imb}\left(= e^{imb}\left(\tilde{J}^{free}\right)\right)$ must be due to free mobility.

First, observe that

$$v_k\left(\cdot,\ \tilde{e}_k^{imb}\right) < v_1\left(\cdot,\ e_1^{free}\right). \tag{9}$$

By construction, the equilibrium with $\tilde{e}^{imb}$ is not in $\mathcal{E}^{free}$. Thus (6) must be violated with $\tilde{e}^{imb}$ so that $v_k\left(\cdot,\ \tilde{e}_k^{imb}\right) \neq v_1\left(\cdot,\ e_1^{free}\right)$. Moreover, starting from $\tilde{J}^{free}$ in $\mathcal{P}^{imb}$, to return to $\mathcal{E}^{free}$ in $\mathcal{P}^{free}$, the utility level in Knoxville should induce an *outflow* of workers, i.e., it should be *lower* than the mobile equilibrium level $v_1\left(\cdot,\ e_1^{free}\right)$, and vice versa for city $k+1$. Assuming an equally weighted social welfare function, the difference between $\mathcal{P}^{imb}$ and $\mathcal{P}^{free}$ amounts to

$$\left(J_k^{free}+\varepsilon\right)v_k\left(\cdot,\ \tilde{e}_k^{imb}\right) - J_k^{free}v_k\left(\cdot,\ e_k^{free}\right) + \left(J_{k+1}^{free}-\varepsilon\right)v_{k+1}\left(\cdot,\ \tilde{e}_{k+1}^{imb}\right) - J_{k+1}^{free}v_{k+1}\left(\cdot,\ e_{k+1}^{free}\right), \tag{10}$$

which may or may not be negative. We will focus on Knoxville to address the positive role that free mobility serves in $\mathcal{P}^{free}$.

Consider how the immobile equilibrium traces back to the mobile equilibrium when free mobility is reinstated and $\tilde{e}_k^{imb}$ returns to e_k^{free}. Let $p_k(e_k)$ denote the equilibrium price vector $(w_k(e_k),\ r_k(e_k))$. We need to know the sign of $\partial e_k^{imb}(J_k)/\partial J_k$ at $J_k = J_k^{free}$ to compare $p_k^{imb}\left(\tilde{e}_k^{imb}\right)$ to $p_k^{free}\left(e_k^{free}\right)$, and corresponding emission levels. However, the conditions (3), (4), (6) and (7) do not tell us the sign without further specifications on utility and production functions. We shall consider all possible scenarios. Let us select five representative emission levels, $e_k^{low} < e_k^{PO} < \tilde{e}_k^{imb} < \bar{e}_k < e_k^{sab}$ in $\mathcal{E}^{free}$.

$$\text{If} \quad \frac{\partial e_k^{imb}(J_k^{free})}{\partial J_k} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \quad \text{then} \quad e_k^{free} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \tilde{e}_k^{imb} \begin{Bmatrix} (\text{e.g., } e_k^{low},\ e_k^{PO}) \\ \\ (\text{e.g., } \bar{e}_k,\ e_k^{sab}) \end{Bmatrix}. \tag{11}$$

Since $\partial e_k^{imb}(J_k^{free})/\partial J_k$ can take any value, pollution abatement is, unfortunately, not unconditional:

Proposition 2.1: Condition for Voluntary Pollution Abatement

Suppose that the economy is in $\mathcal{E}^{imb}$ but not in $\mathcal{E}^{free}$, with population distribution $\tilde{J}^{free}$. By removing restrictions on mobility, city k will see a reduction in emissions if

$$\frac{\partial e_k^{imb}(J_k)}{\partial J_k} > 0. \tag{12}$$

Proof. Immediate from (11). □

The thing is, there are *three* parameters, wage, rent and emission level, that will accommodate the change when free mobility is restored. The emission level does not necessarily have to do all the work. It is useful to have some visual aid representing all three parameters to see why free mobility does not always curb emissions. Define indirect indifference curve $V_k(e_k,\ \bar{v})$ and indirect isoprofit curve $\Pi_k(e_k,\ \bar{\pi})$ as follows:[10]

$$\begin{aligned} V(e_k,\ \bar{v}) &:= \left\{(w_k,\ r_k) \in \mathbb{R}^2_{++}:\ v(w_k,\ r_k;\ Ie_k) = \bar{v}\right\} \\ \Pi(e_k,\ \bar{\pi}) &:= \left\{(w_k,\ r_k) \in \mathbb{R}^2_{++}:\ \pi(w_k,\ r_k,\ e_k) = \bar{\pi}\right\}. \end{aligned} \tag{13}$$

where $\pi(\cdot)$ is an indirect profit function. Figure 1 depicts (13) at different levels of emissions. The diagram is similar to the one that appears in [Rob82], except that her exogenous amenity levels are replaced by endogenous emission levels in figure 1.

Under $\tilde{e}_k^{imb}$ with population $J_k^{free}+\varepsilon$, the immobile equilibrium price is $p_k^{imb}\left(\tilde{e}_k^{imb}\right)$, at which point, $V\left(\tilde{e}_k^{imb},\ v^{imb}\right)$ meets $\Pi\left(\tilde{e}_k^{imb},\ \bar{\pi}\right)$, where $v^{imb} := v\left(w_k\left(\tilde{e}_k^{imb}\right),\ r_k\left(\tilde{e}_k^{imb}\right);\ \tilde{e}_k^{imb}\right)$. The gray dot in figure 1 marks the immobile equilibrium price vector.

As for the mobile equilibrium with e_k^{free} and J_k^{free}, we know from (9) that the mobile equilibrium utility level will be higher than what is achieved with $p_k^{imb}\left(\tilde{e}_k^{imb}\right)$, but we do not know from which $p_k^{free}\left(e_k^{free}\right)$ we arrived at $p_k^{imb}\left(\tilde{e}_k^{imb}\right)$ with perturbation. Figure 1 sketches the indirect indifference and isoprofit curves for each e_k^{free} selected, where $v_1 := v(w_1,\ r_1,\ e_1)$ is the mobile equilibrium utility level with $J = J^{free}$ and $e = e^{free}$ (i.e., without perturbation). Consider the case when $\partial e_k^{imb}(J_k^{free})/\partial J_k = 0$ first. In this case, the emission level e_k^{free} stays at $\tilde{e}_k^{imb}$ and the

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[10] The term 'indirect' indicates that the following are defined over the price of commodities rather than the commodities themselves.

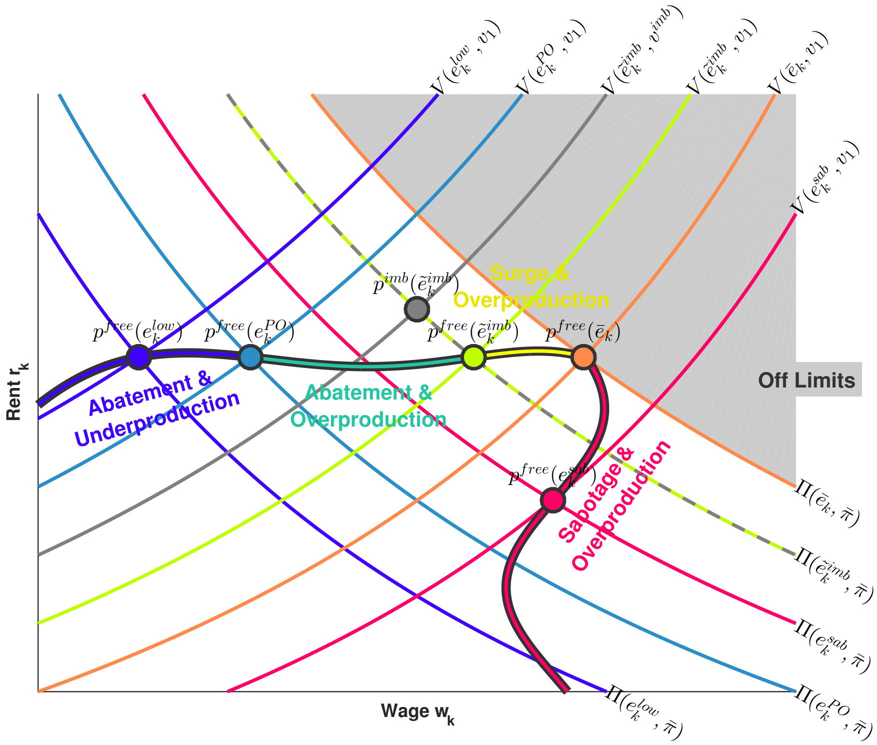

Figure 1.

other two variables will accommodate the change caused by out-migration. When we open the city border, the equilibrium price will simply glide down on the indirect isoprofit curve at $\Pi\left(\tilde{e}_k^{imb}, \bar{\pi}\right)$ (green/gray line in figure 1) to reach $p^{free}\left(\tilde{e}_k^{imb}\right)$ (the green dot in figure 1). As workers move out of Knoxville, the equilibrium wage rises for falling labor supply and the equilibrium rent drops for falling residential housing demand. The wage and rent do all the work in this case and e_k remains neutral — that is, *if* $\partial e_k^{imb}(J_k^{free})/\partial J_k = 0$.

In general, $p_k^{free}\left(e_k^{free}\right)$ can be found anywhere along the bold path in figure 1 depending on the level of $\partial e_k^{imb}(J_k^{free})/\partial J_k$. The change in equilibrium wage and rent caused by a change in J_k (and consequently e_k) is captured through the indirect indifference curve and indifferent isoprofit curve (13). Differentiate $v(\cdot) = \bar{v}$ and $\pi(\cdot) = \bar{\pi}$ to obtain

$$\begin{bmatrix} dw/de \\ dr/de \end{bmatrix} = \frac{1}{\det(A)} \begin{bmatrix} -\pi_r v_e + v_r \pi_e \\ \pi_w v_e - v_w \pi_e \end{bmatrix}, \quad \text{where} \quad A := \begin{bmatrix} v_w & v_r \\ \pi_w & \pi_r \end{bmatrix}. \tag{14}$$

The signs of dw/de and dw/dr depend on whether e_k^{free} is above or below $\bar{e}_k$. Table 1 summarizes the direction of change, from which we can detect where the mobile equilibrium price vector was before perturbation. (Note that $\det(A) < 0$ regardless of the value of e_k, and $\pi_e \gtreqless 0$ for $e_k \lesseqgtr \bar{e}_k$). See appendix A.2 for more on the characteristics of the curves in figure 1.

e_k^{free}	$\left[0,\, e_k^{PO}\right)$	e_k^{PO}	$\left(e_k^{PO},\, \tilde{e}_k^{imb}\right)$	$\tilde{e}_k^{imb}$	$\left(\tilde{e}_k^{imb},\, \bar{e}_k\right]$	$(\bar{e}_k,\, \hat{e}_k]$
example	e_k^{low}	e_k^{PO}		$\tilde{e}_k^{imb}$		e_k^{sab}
dw/de	$+$	$+$	$+$	$+$	$+$	**indeterminate**
dr/de	**indeterminate**	**indeterminate**	**indeterminate**	**indeterminate**	**indeterminate**	$-$
comparison to $\tilde{e}_k^{imb}$	**abatement**	**abatement**	**abatement**	**no change**	**surge**	**sabotage**
comparison to e_k^{PO}	**underproduced**	**correct**	**overproduced**	**overproduced**	**overproduced**	**overproduced**

Table 1. The location of mobile equilibrium price vectors as determined by (14). In the last column, $\hat{e}_k (> 0)$ is a point at which $f(\cdot,\, \hat{e}_k) = 0$. The background color corresponds to the dot and line color in figure 1.

2.5 Pricing out Externalities with Free Mobility

Table 1 indicates that if we trace the equilibrium price vector p^{free} for differing values of $\partial e_k^{free}\left(J_k^{free}\right)/\partial e_k^{free}$ from $p^{free}\left(e^{low}\right)$ on figure 1, the path always travels east towards $p^{free}(\bar{e}_k)$ and then turns south past $p^{free}(\bar{e}_k)$. On the eastbound portion, the equilibrium r_k may go up or down while the equilibrium w_k steadily grows till we climb up to $p^{free}(\bar{e}_k)$ on the equilibrium price path. Past this point, on the southbound portion as we move higher towards $p^{free}\left(e_k^{sab}\right)$, the relationship flips: Now it is the equilibrium w_k that becomes indeterminate and the equilibrium r_k progressively declines with e_k. Indeterminacy and the role reversal stem from the fact that both Josh and Ironworks are on the *same* demand side vying for land whereas they are on the *opposite* side when it comes to labor: Josh is on the supply end of the labor market and Ironworks is on the demand end.

This indeterminacy leaves room for multiple possibilities to reach e^{free} from $\tilde{e}^{imb}$ and raise the utility level to $v_1\left(\cdot,\, e_1^{free}\right)$. The first way is, as we discussed earlier, moving from the gray dot $p^{imb}\left(\tilde{e}_k^{imb}\right)$ in figure 1 to the green dot $p^{free}\left(\tilde{e}_k^{imb}\right)$ (in Josh's favor) without changing the emission level.

The second possibility is to move from $p^{imb}\left(\tilde{e}_k^{imb}\right)$ to somewhere west of $p^{free}\left(\tilde{e}_k^{imb}\right)$ on the bold path in figure 1. As workers leave Knoxville, the wage will drop (cf. (14)) and the utility level will decline but curtailed emissions will more than make up for it when we reach the mobile equilibrium. (The rent may or may not change in this case). Ironworks will incur some productivity loss due to reduced emissions but the cost savings from the reduced wage will offset the productivity loss. In this case, externalities are in part priced out by free mobility as Ironworks find it optimal to lower their emission level in exchange for reduced labor cost till labor outflow stops.

The third way is to move in the opposite direction along the path. Consider the segment tagged "Surge & Overproduction" in figure 1 first. As workers move away, the equilibrium wage will grow while the emission level also grows. For Josh, the benefit of an increased wage outweighs the effect of higher emission levels. Thus, the equilibrium utility level will rise to $v_1(\cdot,\ e_1^{free})$. Ironworks will have the same profit level throughout the change for the same reason as above but in reverse. In fact, at the opposite extreme, there is a far-flung but nonetheless theoretically possible scenario where Ironworks knowingly pumps up e_k above $\bar{e}_k$ in a mobile economy. Since higher e_k can potentially lower the equilibrium rent, which in turn brings down Ironworks' rent payments, they may find it profitable to *increase* their emissions above $\bar{e}_k$ on purpose. Their productivity suffers from too high a level of emissions, but it will be worth it if they can make the air quality so deteriorated that the landlord starts to dump his land on whomever signs a lease for pennies on the dollar. This scenario corresponds to the red "sabotage" segment in figure 1. Not so many people would live in this seriously polluted version of Knoxville but Ironworks will pay almost nothing for their production site.[11] The sabotage situation is possible only in the land market. In the labor market, a high e_k means a high equilibrium wage (recall $w'(e_k) > 0$ for $e_k < \bar{e}_k$ from (14) and table 1) and Ironworks will not save on labor cost by deliberately setting e_k higher as they can through the land market. We will discuss more on the third possibility in section 3.

In any case, opening the city border will make residents in Knoxville better off, but a different adjustment process comes with different implications for the resulting emission level, and thus emission abatement is not guaranteed as we saw in proposition 2.1.

The mobile emission level e^{free} depends on preferences and technologies, and ultimately, on the sign of (12). In this regard, free migration may be viewed as a double-edged sword. As long as the right side is up, it can be used to reduce overproduction of negative externalities. It is actually possible to end up *under*producing negative externalities. Free mobility will *reduce* the equilibrium production level of emissions if (12) is met, but we cannot tell *by how far* it would reduce the emission level. We will see if or when free mobility achieves efficiency in the next segment.

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[11] This may well depend on how the equilibrium wage responds to such a change. The disruptive behavior of the firms may be mitigated by the free-rider problem. See appendix A.1.

2.6 PARETO OPTIMALITY

We will continue our focus on Knoxville. Let us first identify a Pareto optimal allocation before we compare it to $\mathcal{E}^{free}$ and $\mathcal{E}^{imb}$. Consider the following problem:

$$\max_{c_k,\ \tilde{h}_k,\ c_k^L,\ y_k,\ l_k,\ h_k,\ e_k} \quad u\left(c_k,\ \tilde{h}_k;\ Ie_k\right)$$

subject to

$$u^L\left(c_k^L\right) \geq \bar{u}^L \tag{15}$$

$$Iy_k = J_k c_k + c_k^L, \quad J_k = Il_k, \quad H = J_k \tilde{h}_k + Ih_k,$$

$$f\left(l_k,\ h_k,\ e_k\right) \geq y_k,$$

where c_k^L denotes the landlord's consumption and $u^L(\cdot)$ is his utility function. The first order condition with respect to e_k is $\dfrac{\partial u(\cdot)}{\partial e_k} + \lambda_k \dfrac{\partial f(\cdot)}{\partial e_k} = 0$, where λ_k is a Lagrangian multiplier (cf. footnote 8). This leads to

$$\frac{\partial f(\cdot)}{\partial e_k} = \frac{-1}{\lambda_k}\frac{\partial u(\cdot)}{\partial e_k}(> 0). \tag{16}$$

The equilibrium with $\tilde{e}^{imb}$ is Pareto dominated by the one with e^{free} by construction.[12] Thus, between (5) and (8), the mobile equilibrium condition (8) has the potential to coincide with condition (16) for Pareto optimality but its immobile counterpart (5) does not. If, furthermore, (8) does match (16), i.e.,

$$w_k'(E)\frac{J_k(E)}{I} + r_k'(E)h_k\left(w_k(E),\ r_k(E),\ e_k\right) = \frac{-1}{\lambda_k}\frac{\partial u(\cdot)}{\partial e_k} \tag{17}$$

then the mobile equilibrium is efficient. Free mobility is a necessary condition for efficiency as it narrows down a set of candidate allocations for efficiency. In particular, it eliminates $\mathcal{E}_k^{imb} \backslash \mathcal{E}_k^{free}$. We sketched a case when (17) is satisfied and marked the emission level in this case by e_k^{PO} in figure 1.[13] In general, we have

PROPOSITION 2.2: CONDITION FOR VOLUNTARY CORRECTION

If city k's population is $\tilde{J}_k^{free}$ *in* $\mathcal{E}^{imb}$*, free migration leads to*

$$\left\{\begin{array}{c} \textit{overproduction of } e_k \\ \textit{efficient level } e_k^{PO} \\ \textit{underproduction of } e_k \end{array}\right\} \quad \textit{iff} \quad w_k'(E)\frac{J_k(E)}{I} + r_k'(E)h_k\left(w_k(E),\ r_k(E),\ e_k\right) \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} \frac{-1}{\lambda_k}\frac{\partial u(\cdot)}{\partial e_k}. \tag{18}$$

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[12] Insofar as Knoxville is concerned. The same relationship between $\tilde{e}^{imb}$ and $\tilde{e}^{free}$ carries over to the national level as long as (10) is negative.

[13] The location of $p^{free}\left(e_k^{PO}\right)$ is arbitrary and may well be somewhere else. The following arguments are the same regardless.

Proof. Immediate from concavity in assumption 2.1 and (17). ☐

It is then possible that the economy was in the blue "Abatement & Underproduction" portion on the path in figure 1 before perturbation. In this case free mobility goes too far: Ironworks in a mobile economy will overcorrect their emission level in exchange for a significantly lowered equilibrium wage as the blue dot $p^{free}\left(e_k^{low}\right)$ as figure 1 shows. However, even on the rare occasion when e_k^{free} was at a sabotage level e_k^{sab} in $\mathcal{P}^{free}$, the mobile equilibrium still Pareto dominates the immobile equilibrium because of (9). We are not claiming that free mobility will bring about the efficient outcome *for sure*. Rather, the point we raise here is that it *can* price out externalities, sometimes overshooting the target (as in $p^{free}\left(e_k^{low}\right)$) and sometimes undershooting the target (as in "Abatement & Overproduction" portion in figure 1), an option that is *not even possible* in $\mathcal{P}^{imb}$.

With this cautionary observation in mind, let us propose one possible solution to combat negative externalities in the next section.

3 POLICY IMPLICATIONS

Unlike the conventional fixes listed in section 1, which promise to reduce overproduction of negative externalities, our proposed "fix" does not particularly claim to do that. Instead, we have simply shown that *it could be worse* without free mobility. While the historical focus of the literature has been on the control of the direct cause of externalities itself (except for [Tie56]), free mobility has been quietly working behind the scene to cut efficiency loss. We should probably give this unsung hero (free mobility) credit for its unnoticed and unappreciated effort in pollution abatement.

While our claim sounds somewhat passive and pessimistic, one active policy implication that we can draw from the model is to promote labor mobility at the federal and/or local level. We assumed full mobility on the part of workers in section 2 so that the economy *is* $\mathcal{P}^{free}$ rather than $\mathcal{P}^{imb}$. In reality though, workers are not perfectly mobile and our economy is *not exactly* $\mathcal{P}^{free}$. Relocation is usually a costly decision to make. Thus, we still have some room for improvement. In particular, we can turn our economy closer to $\mathcal{P}^{free}$ than to $\mathcal{P}^{imb}$ with appropriate policy measures.

3.1 FEDERAL POLICY

Footloose workers put pressure on firms in pollutant-laden cities to reduce their emission levels, provided the condition in proposition 2.1 is met, or better yet, bring out the efficient outcome if, in addition, (18) holds with equality. Any federal policy that makes it easier for workers to relocate will help make that happen.

Such a policy may backfire on the cities that already have functioning emission regulations in place though. Let us consider the intercity aspect of voluntary pollution abatement (proposition 2.2).

We essentially focused our analysis on one particular city k to establish proposition 2.1 that, in comparison to an immobile equilibrium, a mobile equilibrium can bring about better allocations as it reduces efficiency loss created by overproduction of negative externalities.

It is not certain if the redistribution of residents will make *everyone* (not just Knoxvillians) better off as the sign of (10) can be positive. Consider, for example, that there are 999 identical Knoxvilles and one city, say Louisville, where everything is same as in Knoxville except the emission level e_l, which is already contained at e_l^{PO} in $\mathcal{P}_l^{imb}$ through conventional methods other than mobility. Suppose that the mobile equilibrium is in the abatement portion in figure 1. Upon turning into $\mathcal{P}^{free}$ through some federal policy, Knoxvillians will experience a lower emission level and the new mobile allocation thus updated will Pareto improve upon the immobile allocation they had before. Louisvillians, on the other hand, will see an increase in pollution level, as leaving e_l at e_l^{PO} will set off an influx of people into Louisville and is therefore not sustainable. In this case, as long as cities are identical (except for the difference between e_k and e_l), the benefit that free mobility brings to Knoxvillians may outweigh the damage inflicted upon Louisvillians (or perhaps it may not, if pollution reduces the utility level in a non-linear manner or the social welfare function gives more weight to Louisvillians than to Knoxvillians). Several attempts have been made in the city-size literature (Fujishima [Fuj13] and Tabuchi [Tab82]) to identify the optimal city-size distribution. Our model can be merged with them to see the welfare effect of free mobility, as cities come with a whole array of externalities, some of which are firm-oriented like ours, but some of which are not at a single firm's discretion, such as congestion or economies of urbanization. One thing we can say about the city-size models that feature free mobility and externalities such as Eeckhout [Eec04] is that their equilibrium could have been worse for overpopulated cities if free mobility was taken away.

3.2 Local Policy

Turning to a local level, take cities like Beaumont, TX for example, where residents suffer from noxious fumes from petrochemical plants. The municipal administrators in such cities may actually redirect their current emission control efforts to a more resident-oriented solution. Beaumont officials can turn the tables and make it easier for their *residents* to move out of the city instead of trying to impose direct emission restrictions on the *firms*. What we suggest is for the municipal government to provide extra impetus towards full geographical mobility of workers. Beaumont can reallocate their budget allotted for controlling and monitoring emissions to labor mobility assistance measures like a job search center (for job opportunities *outside* Beaumont, not within Beaumont) or relocation cost subsidies for households (for residents moving *out of* Beaumont, not into Beaumont). If everything goes well, the petrochemical plants should pick up the rising labor cost and readjust their emission levels. In a sense, Beaumont city administrators are holding their residents hostage to strike a better deal with the plants, namely, a better equilibrium wage and/or lower emissions (hostage in the sense that workers will be off limits to the plants unless the plants pay a decent wage). Workers have competitive outside options readily available out of town thanks to a newly implemented policy. By reducing relocation costs, Beaumont is helping their residents to (literally) walk away from the plants overproducing negative externalities. Of course Beaumont does not negotiate with the plants by holding up the residents, but the "negotiation" is conducted silently through the market equilibrium in $\mathcal{P}^{free}$, which used to be rather close to $\mathcal{P}^{imb}$ before the introduction of relocation assistance. Thus, there is no negotiation cost with this mobility policy as labor and land markets will take care of the pricing. In essence, Beaumont will be circulating *workers* rather than *pollutants* to manage the city's air quality problem with this mobility solution.

A problem with this approach is that 1) since firms are not identical, the policy may complicate a free-rider problem (cf. appendix A.1), and 2) it can also backfire if the inequality in (12) turns out to be in reverse and the sabotage case (the red segment in figure 1) takes place instead. The outcome is going to be disastrous, or apocalyptic even, especially if 2) happens because the end result is autonomous with our solution. Recall that if e_k^{free} turns out to be above $\bar{e}_k$, the equilibrium rent will drop (cf. (14) and table 1). Beaumont will wind up with dirt-cheap land with emission control going out of the window to keep the rent low, and with few residents left in town, who enjoy the low rent but may be paid even less in equilibrium (there is no guarantee that the equilibrium wage is higher in $\mathcal{P}^{free}$ than in $\mathcal{P}^{imb}$ when $e_k^{free} > \bar{e}_k$ according to (14) and table 1). The utility level is still higher in $\mathcal{P}^{free}$ (from (9)) but the number of people who enjoy the said level of utility will be very small.

Any fixes listed in section 1 are subject to miscalculations but ours is even more vulnerable to oversights. Once the economy reaches the equilibrium, however high the equilibrium e_k^{free} in $\mathcal{P}^{free}$ turns out to be, there is no push-back to the original equilibrium in $\mathcal{P}^{imb}$. The economy will stay wherever it reached. Mobility aside, e_k^{imb} and e_k^{free} are both an equilibrium value after all: They emerge of their own accord. By the time Beaumont realizes $e_k^{free} > \bar{e}_k$, it will probably be too late to reverse their actions. The most important variable in this policy is mobility and that is the only weapon Beaumont has in our scenario. The city can reverse mobility assistance measures by making it hard to move out of the city and return to $e_k^{imb}\left(< e_k^{free}\right)$. But to do so, they need to regain the previous level of population they had in $\mathcal{P}^{imb}$. Now that the city's pollution level is way over the top, it is hard to imagine anyone willingly moving into this severely polluted city to the rescue. Thus, extra caution should be exercised with our suggested fix as the end result could be irreparable if the adopted policy backfires.

4 Conclusion and Extensions

We introduced mobile and immobile production economies with externalities, and established that under certain conditions, externalities can be internalized when workers are perfectly mobile. Our premise is that a firm will recognize the cost of externalities of their making indirectly through the market equilibrium. Furthermore, in a mobile economy, a high emission level needs to come with a high equilibrium wage (and potentially but not necessarily with a low equilibrium rent), or else, workers will walk away from the city. Thus, free mobility forces the firm to own up to the damage they inflict on the city's environment in the form of a high labor cost.

While it can reduce the emission level, the mobile equilibrium may not necessarily be efficient. It has a potential to bring about the efficient outcome, but that depends on how sensitive the equilibrium wage and rent are to the emission level, and ultimately on the condition in proposition 2.1. We know for certain, though, that our equilibrium would have been worse if mobility is restricted.

We built our model on [Rob82]. $\mathcal{P}^{free}$ can be thought of as a special case of her model where e_k is restricted to the profit-maximizing level rather than any random level. Similarly, [Tie56] can be thought of as a different version of our model where e_k is still endogenous but implemented by a public sector rather than firms, and externalities are limited to local public goods.

The implication of our model is that if labor mobility is constrained, federal and/or local governments can take measures to increase labor mobility by subsidizing moving expenses or reducing the cost of job search outside the city. However, the policy may backfire and lead to a catastrophic result if the government miscalculates how the equilibrium wage and rent respond to negative externalities, and overshoots the value of (12). Our suggested measures need to be implemented with caution.

We conclude the paper with two remarks. First, we did not explicitly examine the case for *positive* externalities, but that can be done by simply flipping the sign of e_k.

Second, we restricted our focus on the externalities that stay within the city border. However, some externalities may travel across the border. For instance, pungent odor from a paper mill located in other city may reach your city on a windy day. Contaminated water produced in a city upstream from your city will adversely affect your quality of life. An accident at a nuclear power plant can have disastrous consequences over a wide range of areas. The effect of long-range externalities in the presence of free mobility may be another topic to investigate. Unfortunately, it is likely that our suggested abatement will be limited because firms in Knoxville do *not* share the equilibrium wage or rent with Louisville. In fact, for public good provision, Calabrese et al. [CER12] have shown that Tiebout's efficient outcome falls apart in such cases. The same goes for our case even when Knoxville underproduces pollutants in $\mathcal{P}^{free}$ (the blue portion in figure 1). Pollutants from other cities will raise e_k and it may reach e_k^{PO}. However, e_k^{PO} is determined within Knoxville and the only medium of intercity interactions in $\mathcal{P}^{free}$ is designed to be out-of-town workers, not out-of-town pollutants. If the benefit of increased e_k goes to firms outside Knoxville, then e_k^{PO} thus reached will not be efficient. Surrounding cities are responsible for the damage forced upon Knoxvillians but they do not pay for it.

A Appendix

A.1 Free-Rider Problem

Since we consider a general equilibrium model, it is inconclusive whether we encounter a free-rider problem in our economy. First, note that each e_k enters into E_k with the same weight (cf. the second assumption in assumption 2.1). Thus, firms' first order conditions (5) and (8) are the same for any firm. Ironworks' actions affect other firms just as much as they affect Ironworks itself.

Suppose that the economy is in equilibrium with $e_k > e_k^{sab}$ for example. Ironworks can intentionally lower its own emission levels while letting other firms go overboard on emissions. Then Ironworks will enjoy a low rental cost while increasing its productivity on the back of other firms. But then when Ironworks reduces its emission levels, rent reduction will weaken because e_k will be marginally smaller due to the lack of Ironworks' "contribution". Consequently, they will not capture the full cost advantage from excessive emissions, and if all the other firms follow suit, the rent reduction effect will completely disappear and everyone will end up failing to meet the first order condition. Thus, we will encounter a free-rider problem — that is, *if* the equilibrium wage remains the same throughout.

Since the equilibrium wage also responds to e_k and its response is indeterminate above $\bar{e}_k$ (cf. table 1), it is not certain if the firms will be trapped in such a prisoner's dilemma. If $w'(e_k) < 0$ as well as $r'(e_k) < 0$ above $\bar{e}_k$, then Ironworks will want to let other firms do the dirty work. If not, Ironworks may end up losing its profit by not setting e_k above $\bar{e}_k$ as its labor cost increase may exceed rent savings. The same goes for the case when $e_k < \bar{e}_k$ with the role of wage and rent flipped. Therefore, if the equilibrium price path runs to the southeast or to the northwest of $p^{free}(\bar{e}_k)$ on figure 1, the free-rider problem may not happen after all.

A.2 INDIRECT INDIFFERENCE AND ISOPROFIT CURVES

Some caveats are in order regarding the two curves (13). While $\Pi(e_k,\ \bar{\pi})$ for $e_k > \bar{e}_k$ is well defined, at first glance, it is baffling that it even exists. Take $e_k^{sab}(> \bar{e}_k)$ in figure 1 for example. If Ironworks is currently on $\Pi(e_k^{sab}, \bar{\pi})$ (the red line), it can turn a higher profit than $\bar{\pi}$ by simply cutting back on e_k. Then how can Ironworks end up with the *same* profit level $\bar{\pi}$ both at $\bar{e}_k$ *and* e_k^{sab}?

That would be a sensible question to ask if emission levels are exogenous as in [Rob82]. However, a careful examination should reveal that any deviation from $\bar{e}_k$, be it upwards (e_k^{sab}) or downwards (e_k^{PO}, or e_k^{low}), comes with a change in $(w_k,\ r_k)$ as we discussed in section 2.2. Ironworks can earn the same profit $\bar{\pi}$ even when it chooses different emission levels because e_k does not only change its revenue but also its cost. A profit maximizing e_k may not be the same as the output maximizing $e_k = \bar{e}_k$.

On a related matter, since Ironworks' revenue is highest at $\bar{e}_k$, any departure from $\bar{e}_k$ should be accompanied by cost advantages through a lower equilibrium wage and/or rent if Ironworks stays at the same profit level $\bar{\pi}$. This furthermore means that for any given $\bar{\pi}$, $\Pi\left(e_k,\ \bar{\pi}\right)$ will run below $\Pi\left(\bar{e}_k,\ \bar{\pi}\right)$ for all $e_k \in \mathbb{R}_+$. In particular, if $e_k > \bar{e}_k$, at least one[14] of w_k or r_k needs to be lower than the ones on $\Pi\left(\bar{e}_k,\ \bar{\pi}\right)$ to keep to $\bar{\pi}$ as e_k reduces productivity beyond $\bar{e}_k$. Thus, $\Pi\left(\bar{e}_k,\ \bar{\pi}\right)$ is the upper envelope of all the indirect isoprofit curves at $\bar{\pi}$. Any $(w_k,\ r_k)$ above $\Pi\left(\bar{e}_k,\ \bar{\pi}\right)$ (blocked out area tagged "off limits" in figure 1) may never be realized at $\bar{\pi}$.

Consequently, unlike regular indifference curves, two distinct indirect isoprofit curves $\Pi\left(e_k^{low},\ \bar{\pi}\right)$ and $\Pi\left(e_k^{high},\ \bar{\pi}\right)$ with $e_k^{low} < \bar{e}_k < e_k^{high}$ can share the same point $(w_k,\ r_k)$ and cross each other. For example $\Pi(e^{sab},\ \bar{\pi})$ can go across any of the indirect isoprofit curves $\Pi(\tilde{e}_k^{imb},\ \bar{\pi})$, $\Pi(e_k^{PO},\ \bar{\pi})$, or $\Pi(e_k^{low},\ \bar{\pi})$ on figure 1. The same does not go for the indirect indifference curve as $v(\cdot)$ is *monotone decreasing* in e_k.

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[14] Notice that we did not say "both" here. The shaded area labeled "off limits" in figure 1 only cuts off the northeastern portion of the plane. The area to the southeast or the northwest of $p\left(e^{imb}\right)$ is not completely shut out. Proposition 2.2 explains why.

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